

Galileons, phantom fields, and the fate of the Universe

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In this paper we study cosmological dynamics of phantom as well as non-phantom fields with linear potential in presence of Galileon correction $(\partial_\mu\phi\partial^\mu\phi)\Box\phi$. We show that the Big Crunch singularity is delayed compared to the standard case; the delay crucially depends upon the strength of Galileon correction. As for the phantom Galileon, ρ_ϕ is shown to grow more slowly compared to the standard phantom delaying the approach to singularity. In case, $V \sim \phi^n, n > 4$, Big Rip is also delayed, similar phenomenon is shown to take place for potentials steeper than the exponential.

I. INTRODUCTION

Observations in cosmology have recently led to confirmation that the Universe is undergoing an accelerated phase of expansion at present [1, 2]. The direct support for the phenomenon came from the observations of supernovae of type Ia (SNe Ia) [1]. The explosions of these SNe Ia look fainter than expected in the Einstein de-Sitter model. The concept of “*dark energy*” was introduced to explain the luminosity-redshift observations of these type Ia supernovae by modifying the right hand side of Einstein field equations which give rise to an accelerated expansion of the Universe and thus explains the unexpected faintness of the supernovae. The weird form of energy yet remains to be mysterious as there is no direct observational test to probe it but this is generally assumed that it has a large negative pressure [3].

In past few years there have been a number of activities for modelling dark energy including the models with the scalar field and brane world etc. To this effect, a large varieties of scalar field models are discussed in the literature including quintessence [4], K-essence [5], spintessence [6], tachyon [7], quintom [8, 9], chameleon [10] and many more. These models of scalar field give the equation of state parameter $w \geq -1$. It is interesting to note that the observational data also allows models of dark energy with equation of state parameter crossing -1 line (called phantom field models). Thus, a number of phantom models have been discussed in the literature [11–15], for instance, brane world and non-minimally coupled scalar field models can give phantom energy [16–18]. The simplest way to introduce the phantom effect is provided by a scalar field with negative kinetic energy term which could be motivated from S-brane constructs in string theory [19]. The concept of phantom field was first used in steady state theory of Hoyle and subsequently incorporated in Hoyle and Narlikar theory of gravitation [20].

The future singularity termed as “Big Rip” [21] naturally arises in models with $w < -1$ and is characterized by the divergence of the scale factor after a finite interval of time. It is generic to keep w as time dependent rather than to consider it as a constant. This choice of w generates specific scalar field models to avoid the cosmic doomsday [12, 22] which requires a particular class of phantom field potentials.

There are alternative ways to explain the accelerated expansion by modifying the left hand side of Einstein field equations *a la* modified theories of gravity. Following this, a special class of dark energy model based on the large scale modification of gravity called *Galileon gravity* [23, 24] was proposed. The distinguished feature of this theory is that it provides a consistent modification of general relativity leaving the local physics intact. This modified gravity in this scheme can give rise to the observed late time cosmic acceleration and also it is free from negative energy instabilities. The Galileon field has five field Lagrangians \mathcal{L}_i ($i = 1, \dots, 5$) in 4-dimensional space-time. The Lagrangian \mathcal{L}_1 is linear, \mathcal{L}_2 is the standard kinetic term and \mathcal{L}_3 represents the Vainshtein term consisting of three Galileon fields that is related to the decoupling limit of Dvali, Gabadadze, and Porrati (DGP) model [25] while \mathcal{L}_4 and \mathcal{L}_5 consists of higher order non-linear derivative terms of field. In case, we study scalar field with linear potential, it becomes obligatory to compliment it by the higher derivative Galileon terms in the Lagrangian. For simplicity, we shall consider the lowest Galileon term \mathcal{L}_3 for phantom and non-phantom fields with linear potential. On purely phenomenological grounds, we also examine the phantom case with a general potential term $V(\phi)$ [26–28] complimented by Galileon term. In this case, we focus on some general features of cosmological dynamics, in particular, current acceleration and future evolution of the Universe.

Recently, it was found that in quintessence models where scalar field potentials turn negative might lead to collapse of the Universe in distant future [29–33] dubbed Big Crunch singularity. Lykkas

and Perivolaropoulos show that the cosmic doomsday singularity can be avoided in Scalar-Tensor Quintessence with a linear potential by some values of the non-minimal coupling parameter [34]. In this paper, we shall examine these and other aforesaid issues in presence of Galileon correction \mathcal{L}_3 in the Lagrangian.

The paper is organized as follows. In Section II, we consider Galileon field model with linear potential which is generically non-minimally coupled scalar field model and investigates present and future evolution of the Universe for both phantom and non-phantom cases. In Section III, we consider Galileon phantom field model with steep exponential potential and examine the future evolution of the Universe. We summarize our results in the Section IV.

II. GALILEON FIELD WITH LINEAR POTENTIAL

In this section, we consider Galileon field action possessing up to the third order term in the Lagrangian with a field potential $V(\phi)$ [26–28].

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{pl}^2}{2} R - \frac{1}{2} \epsilon (\nabla \phi)^2 - \frac{\beta}{2M^3} (\nabla \phi)^2 \square \phi - V(\phi) \right] + S_m \quad (1)$$

where, $\epsilon = -1$ and $+1$, for phantom and non-phantom Galileon fields respectively. $M_{pl}^2 = 1/8\pi G$ is the reduced Plank mass and the constant β is dimensionless. S_m entitles the matter action and M is a constant of mass dimension one. For simplicity, we fix here $M = M_{pl}$. In a homogeneous isotropic flat Friedmann-Lemaitre-Robertson-Walker (FLRW) Universe, the equations of motion are obtained by varying the action (1) with respect to metric tensor $g_{\mu\nu}$ and scalar field ϕ as

$$3M_{pl}^2 H^2 = \rho_m + \frac{1}{2} \epsilon \dot{\phi}^2 - 3 \frac{\beta}{M_{pl}^3} H \dot{\phi}^3 + V(\phi), \quad (2)$$

$$M_{pl}^2 (2\dot{H} + 3H^2) = -\frac{1}{2} \epsilon \dot{\phi}^2 - \frac{\beta}{M_{pl}^3} \dot{\phi}^2 \ddot{\phi} + V(\phi), \quad (3)$$

$$\frac{\ddot{a}}{a} = -\frac{1}{6M_{pl}^2} \left(\rho_m + 2\epsilon \dot{\phi}^2 - \frac{3\beta}{M_{pl}^3} (H \dot{\phi}^3 - \dot{\phi}^2 \ddot{\phi}) - 2V(\phi) \right), \quad (4)$$

$$\epsilon \ddot{\phi} + 3H \epsilon \dot{\phi} - 3 \frac{\beta}{M_{pl}^3} \dot{\phi} (3H^2 \dot{\phi} + \dot{H} \dot{\phi} + 2H \ddot{\phi}) + V'(\phi) = 0, \quad (5)$$

where, $\dot{}$ denotes derivative with respect to ϕ and ρ_m is the energy density of matter. The energy conservation equation of matter is given by

$$\dot{\rho}_m + 3H \rho_m = 0. \quad (6)$$

In the radiation/matter dominated phase, the Universe is dominated by a perfect fluid with equation of state $p = w\rho$. In this phase of evolution the density of matter ρ_m dominates over the field ϕ . With the expansion of the Universe over time the Hubble parameter H begins decreasing and the scalar field ϕ starts dominating. The total energy content of the Universe $\rho_{total} = \rho_m + \rho_\phi \simeq \rho_\phi = \frac{1}{2} \epsilon \dot{\phi}^2 + V(\phi) - \frac{3\beta}{M_{pl}^3} H \dot{\phi}^3$. Therefore equation (2) reduces to

$$3M_{pl}^2 H^2 = \frac{1}{2} \epsilon \dot{\phi}^2 + V(\phi) - \frac{3\beta}{M_{pl}^3} H \dot{\phi}^3, \quad (7)$$

which is difficult to solve analytically. In what follows, we shall solve the evolution equations numerically and plot the future evolution graphically.

Introducing the dimensionless parameters

$$\begin{aligned} H_0 t &\longrightarrow t, \\ \frac{\phi}{\sqrt{3}M_{pl}} &\longrightarrow \phi, \\ \frac{V_0}{\sqrt{3}M_{pl}^2 H_0^2} &\longrightarrow V_0. \end{aligned} \quad (8)$$

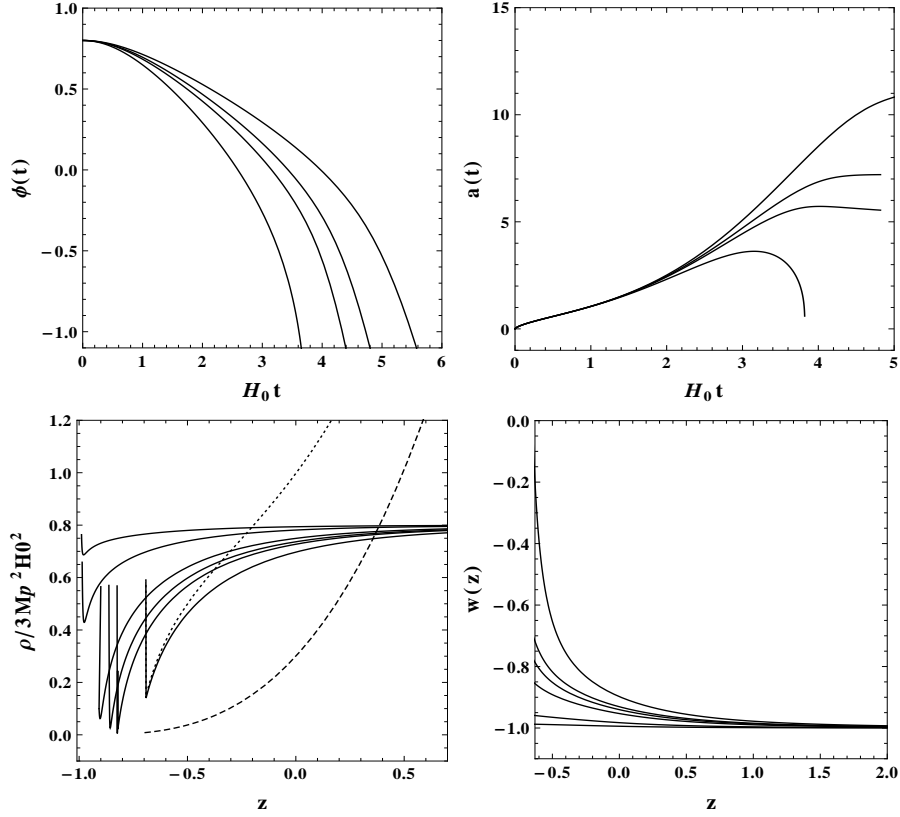


FIG. 1: The upper panels show the evolution of field $\phi(t)$ and scale factor $a(t)$ versus time ($H_0 t$) for Galileon field having linear potential for different values of β and $V_0 = 1$ showing collapse nature in future (see upper right panel). The time is normalized by H_0 (Hubble constant at present epoch). The present time corresponds to $t_0 = 0.96$. The left bottom panel represents the evolution of energy density ρ versus redshift z . The solid lines correspond to Galileon field for different values of β . The dashed and dotted lines represent the energy density of matter and total energy density of the Universe respectively with $\beta = 0$ (quintessence). The right bottom panel shows the evolution of equation of state parameter w versus redshift z for Galileon field with different values of β . In this figure, upper panels are plotted for $\beta = 0, 0.3, 0.5, 1$ (for more higher values, collapse shifted in far distant future) whereas the lower panels have $\beta = 0, 0.3, 0.5, 1, 10, 100$ from bottom to top but in the bottom right panel from top to bottom.

The system of equations (4) and (5) can be written as

$$\frac{\ddot{a}}{a} = -\epsilon\dot{\phi}^2 + \frac{3\sqrt{3}\beta}{2} \left(\frac{H_0}{M_{pl}} \right)^2 \left(\frac{\dot{a}}{a} \dot{\phi}^3 - \dot{\phi}^2 \ddot{\phi} \right) + V_0 \phi - \frac{\Omega_{m0}}{2a^3}, \quad (9)$$

$$\epsilon\ddot{\phi} + 3\frac{\dot{a}}{a} \epsilon\dot{\phi} - 3\sqrt{3}\beta\dot{\phi} \left(\frac{H_0}{M_{pl}} \right)^2 \left\{ 3\frac{\dot{a}^2}{a^2} \dot{\phi} + \left(\frac{a\ddot{a} - \dot{a}^2}{a^2} \right) \dot{\phi} + 2\frac{\dot{a}}{a} \ddot{\phi} \right\} + V_0 = 0. \quad (10)$$

The equation of state parameter w for Galileon field is defined as

$$\begin{aligned} w &= \frac{p_\phi}{\rho_\phi}; \\ p_\phi &= \frac{1}{2} \epsilon\dot{\phi}^2 + \frac{\beta}{M_{pl}^3} \dot{\phi}^2 \ddot{\phi} - V(\phi), \\ \rho_\phi &= \frac{1}{2} \epsilon\dot{\phi}^2 - 3\frac{\beta}{M_{pl}^3} H \dot{\phi}^3 + V(\phi). \end{aligned} \quad (11)$$

In case of Galileon field model, we are considering two cases phantom and non-phantom. First we shall discuss non-phantom case.

Case I: Non-phantom ($\epsilon = +1$)

When $\beta = 0$ the Galileon field action (1) reduces to the standard quintessence field. In equations (9) and (10), we have two variables, namely, scale factor a and field ϕ . The term $3\beta H\dot{\phi}^3/M_{pl}^3$ in equation (2) is the Galileon correction term which depends upon a , ϕ and parameter β . The different values of β puts the strength of Galileon correction term over the quintessence term. For $\beta = 0$, the evolution of Galileon field model is same as the standard quintessence throughout the history of the Universe. Hence all non zero values of β find the departure from quintessence and also the effect of Galileon correction term. Therefore in this analysis we take β as a model parameter.

Now we solve the equations (9) and (10) numerically with the assumption that the field ϕ was frozen initially (i.e. $\phi(t_i) = \phi_i$ and $\dot{\phi}(t_i) = 0$) caused by huge Hubble damping. This is identical to thawing type of models [35]. We use following initial conditions ($t \rightarrow t_i \simeq 0$)

$$\begin{aligned} a(t_i) &= \left(\frac{9\Omega_{0m}}{4}\right)^{1/3} t_i^{2/3} \\ \dot{a}(t_i) &= \frac{2}{3} \left(\frac{9\Omega_{0m}}{4}\right)^{1/3} t_i^{-1/3} \\ \phi(t_i) &= \phi_i \\ \dot{\phi}(t_i) &= 0. \end{aligned} \tag{12}$$

With the above initial conditions and by tuning ϕ_i , we get the following parameters at the present time,

$$\begin{aligned} a(t_0) &= 1, \\ H(t_0) &= 1, \\ \Omega_{0m} &= 0.3, \end{aligned} \tag{13}$$

where, t_0 is defined as the time when the scale factor is unity. In the upper panels of figure 1, we present the dynamical evolution of field ϕ and scale factor a for different values of β and $V_0 = 1$. For $\beta = 0$, the evolution of a is alike to the standard quintessence model. Initially, the field is positive and the Universe gets expansion with the late time cosmic acceleration as soon as field changes sign, in future, potential becomes negative and the scale factor collapses to a Big Crunch singularity. However, for larger values of β the sign of field changes in more distant future and correspondingly $V(\phi)$ becomes negative. Therefore collapse of scale factor is shifted in more distant future for higher values of β . In other words, the cosmic doomsday is delayed for $\beta > 0$.

In the lower left panel of figure 1, we show the evolution of energy density for various values of β and $V_0 = 1$. Initially, the Galileon field imitates the Λ CDM like behaviour and its energy density is highly subdominant to the matter energy density ρ_m and persists so, for most of the time of expansion. The Galileon field remains in the state with $w = -1$ till the epoch ρ_ϕ goes near to ρ_m . At late times, the energy density of Galileon field gets to the matter, overtakes it and begins decreasing ($w > -1$), and acquires the present accelerated expansion of the Universe having $\Omega_{0m} \simeq 0.3$ and $\Omega_{0\phi} \simeq 0.7$. Thereafter ρ_ϕ continuously decreases until it comes a point where ϕ is negative (i.e. $\phi < 0$) and $\dot{\phi}^2/2 + V(\phi) - 3\beta H\dot{\phi}^3/M_{pl}^3 = 0$. Therefore, $H \rightarrow 0$, i.e. the total energy density of the Universe reaches to zero and bounce occurs. For $\beta = 0$, Galileon field behaves as standard quintessence and the similar behaviour for quintessence is shown in ref. [33]. As we go for higher values of β ($\beta = 0.3, 0.5, 1, 10, 100$) the bounce and collapse shifted in distant future. One can say that the bounce and collapse delayed for higher values of β .

The evolution of equation of state for $V_0 = 1$ and various values of β is shown in the lower right panel of figure 1. For $\beta = 0$, the equation of state of Galileon field reduces to the equation of state of standard quintessence and diverges from the equation of state of Λ CDM model. As the values of β are increased, we get more and more deviation in w from the case of standard quintessence and approaches towards the Λ CDM model. The higher values of β for Galileon field with linear potential are in good agreement with the observations as shown in ref. [27] where they have imposed observational constraints on Galileon correction term which is associated with β .

Case II: Phantom ($\epsilon = -1$)

First we are considering the case of $\beta = 0$. Hence, the action (1) reduces to the action of phantom field minimally coupled to gravity and matter.

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{pl}^2}{2} R + \frac{1}{2} (\nabla\phi)^2 - V(\phi) \right] + S_m \tag{14}$$

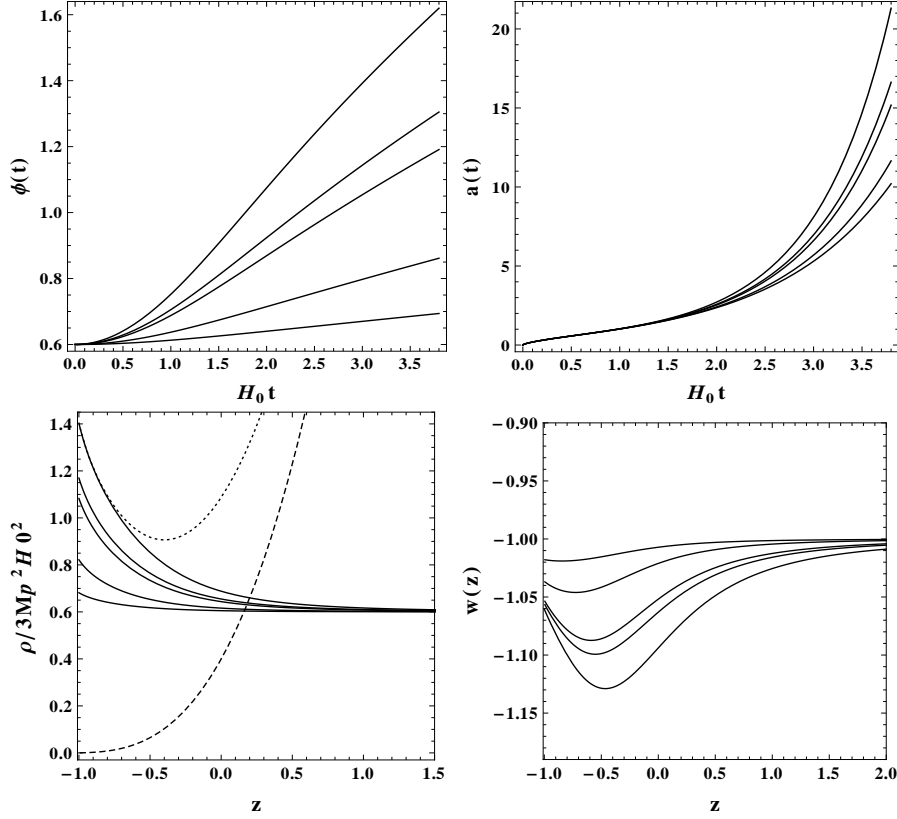


FIG. 2: The evolution of field $\phi(t)$ and scale factor $a(t)$ versus time ($H_0 t$) for Galileon phantom field with linear potential are plotted and shown in the upper panels for different values of β and $V_0 = 1$. The time is normalized by H_0 and the present time is $t_0 = 0.96$. The upper right panel shows the divergent nature of scale factor, in future, after some finite interval of time. The energy density ρ versus redshift z is shown in the left bottom panel, where the solid lines correspond to Galileon phantom field for various values of β . The dashed and dotted lines represent the energy density of matter and total energy density of the Universe respectively for $\beta = 0$ (standard phantom). The right bottom panel shows the evolution of equation of state w versus redshift z for Galileon phantom field with various values of β . In this figure, all the panels have $\beta = 0, 0.5, 1, 10$ and 100 from top to bottom whereas bottom right panel from bottom to top.

The wrong sign in the kinetic energy term of equation (14) gives the ghost field in the context of quantum field theory or phantom field in cosmology. As a dark energy candidate, the equation of state of phantom field is marginally favoured by the present observations [36]. The vital cosmological dynamics of phantom field has been broadly discussed in the literature. However, it is plagued with intense quantum instabilities. Theoretically, we still do not know the basic origin of $w < -1$. In the recent past, it has been discussed that the opposite sign in the kinetic energy term does not give instabilities, required that higher order derivative terms should be included in the action [37].

We take Galileon phantom field model by invoking negative sign in the kinetic energy term. For $\beta = 0$, it behaves as a standard phantom field model. In this case, the initial kinetic term of the phantom field decreases due to Hubble damping term in equation (10) and as a result the field freezes for a while till the epoch ρ_ϕ approaches to ρ_m (see bottom left panel of figure 2). Eventually, the field switches on and the future evolution depends upon the shape of the potential $V(\phi)$.

When $\dot{\phi}$ is nearly frozen and phantom energy starts to dominate, then the system of equations (2) and (5) reduces to (for $\beta = 0$ case)

$$H^2 \simeq \frac{V(\phi)}{3M_{pl}^2}, \quad \dot{\phi} \simeq \frac{V'(\phi)}{3H}. \quad (15)$$

The ratio kinetic to potential term can be written as

$$\frac{\dot{\phi}^2}{2V(\phi)} = \frac{M_{pl}^2}{6} \frac{V'^2}{V(\phi)^2} = \frac{M_{pl}^2}{6} \frac{1}{\phi^2}, \quad (16)$$

where, $V(\phi) = V_0 \phi/M_{pl}$, the ratio kinetic energy to potential energy term is proportional to $1/\dot{\phi}^2$ and goes to zero as a result the kinetic energy term remains sub-dominant continually. This is similar to the slow-roll regime for an ordinary field and can be called as “slow climb” [15, 38]. How to exit from rip was discussed in Ref. [39]. The equation of state approaches towards -1 (see bottom right panel of figure 2) with an increasing energy density as shown in bottom left panel of figure 2. The estimation $\frac{\dot{\phi}^2}{2V(\phi)} \rightarrow 0$ is not valid for an exponential and steeper potentials. We shall discuss this case in Section III. In case of phantom with Galileon correction (for $\beta \neq 0$), with the domination of phantom energy and $\dot{\phi}$ is small, the system of equations (2) and (5) reduces to (by taking the subleading terms)

$$H^2 \simeq \frac{V(\phi)}{3M_{pl}^2}, \quad \dot{\phi} \simeq \frac{M_{pl}^3}{6\beta H} \left[-1 \pm \sqrt{1 + \frac{4\beta}{M_{pl}^3} V'(\phi)} \right]. \quad (17)$$

In slow roll approximation, the term $\frac{4\beta}{M_{pl}^3} V'(\phi)$ is small and we have from (17)

$$\frac{\dot{\phi}^2}{2V} \approx \frac{M_{pl}^2}{6} \left(\frac{V'(\phi)}{V(\phi)} \right)^2 \left[1 - \frac{2\beta}{M_{pl}^3} V'(\phi) \right], \quad (18)$$

showing that the presence of Galileon correction term enhances the slow climb for monotonically increasing $V(\phi)$. Keeping in mind the, $\epsilon = -\frac{\dot{H}}{H^2} = \frac{3}{2}(1 + \omega_{eff}(\phi))$, we have shown numerically that the Galileon correction term for large values of β moves $\omega_{eff}(\phi)$ towards the de-Sitter point, though $\omega_{eff}(\phi)$ yet remains to be less than -1 .

In the upper panels of figure 2, we show the evolution of phantom field ϕ and scale factor a versus time. The phantom field and scale factor both diverges after finite interval of time (in future) and correspondingly energy density of phantom field ρ_ϕ increases slowly (see bottom left panel of figure 2). In this case, the equation of state first decreases from -1 and then eventually increases towards -1 and comes near to it asymptotically [15] that is shown in bottom right panel of figure 2. To this effect, final Universe would be different from both the de-Sitter and Big Rip, and an infinite time would be taken to reach an infinite energy density.

When we add Galileon correction term and go for higher values of β (0.5, 1, 10, 100), the scale factor shows less divergence nature than the case of $\beta = 0$ and correspondingly ρ_ϕ grows slowly as shown in the upper right and left bottom panels of figure 2; Initially, the Galileon phantom field imitates the Λ CDM like behaviour and its energy density is highly sub-dominant to the matter energy density ρ_m and remains so, for most of the time of evolution. The Galileon phantom field remains in the state with $w = -1$ till the epoch ρ_ϕ goes near to ρ_m . At late times, the energy density of Galileon phantom field approaches to matter, overtakes it and begins increasing ($w < -1$), and acquires the present accelerated expansion of the Universe having $\Omega_{0m} \simeq 0.3$ and $\Omega_{0\phi} \simeq 0.7$. For higher values of β ($\beta = 0.5, 1, 10, 100$) the slow growing divergence shifted towards lower values of ρ_ϕ . In the right bottom panel of figure 2, we present the evolution of w versus redshift z for Galileon phantom field. For $\beta = 0$, the equation of state of Galileon phantom field deviates more from the equation of state of Λ CDM model. As the values of β are increased, we get less deviation from Λ CDM. For all values of β , the equation of state first decreases from -1 and then subsequently increases towards -1 and comes near to it asymptotically. Hence, we get smaller deviation in equation of state parameter w from Λ CDM for $\beta > 0$. The effect of the Galileon correction on the evolution of phantom field was also studied in Ref. [40].

III. GALILEON PHANTOM FIELD WITH EXPONENTIAL POTENTIAL

We consider the Galileon phantom field with exponential potential. This is the purely phenomenological case that is just to establish more liberty and workability. However, this type of potential breaks the Galileon shift symmetry. It is alike to most of the phantom field models in which potentials are completely phenomenological. The system of equations (4) and (5) with the equation (8) for an exponential potential can be written as

$$\frac{\ddot{a}}{a} = \dot{\phi}^2 + \frac{3\sqrt{3}\beta}{2} \left(\frac{H_0}{M_{pl}} \right)^2 \left(\frac{\dot{a}}{a} \dot{\phi}^3 - \dot{\phi}^2 \ddot{\phi} \right) + \frac{V_0}{\sqrt{3}} e^{3\phi^2} - \frac{\Omega_{0m}}{2a^3}, \quad (19)$$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + 3\sqrt{3}\beta\dot{\phi} \left(\frac{H_0}{M_{pl}} \right)^2 \left\{ 3\frac{\dot{a}^2}{a^2}\dot{\phi} + \left(\frac{a\ddot{a} - \dot{a}^2}{a^2} \right) \dot{\phi} + 2\frac{\dot{a}}{a}\ddot{\phi} \right\} = 2\sqrt{3}V_0\phi e^{3\phi^2}, \quad (20)$$

where, we have used $V(\phi) = V_0 \exp(\phi^2/M_{pl}^2)$. Now, we solve the system of equations (19) and (20) numerically with the equation (12).

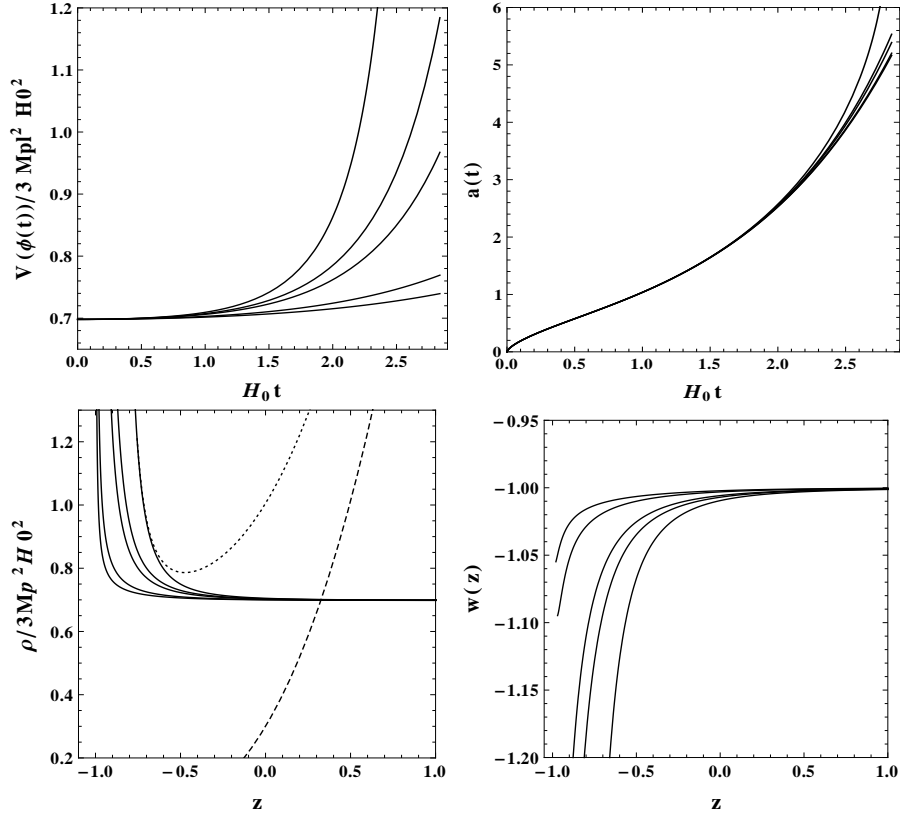


FIG. 3: The upper panels show the evolution of potential $V(\phi(t))$ and scale factor $a(t)$ versus time ($H_0 t$) for Galileon phantom field with exponential potential which is more steeper than the linear potential for different values of β . The scale factor $a(t)$ shows the divergence nature after finite interval of time (in distant future). Here also the time is normalized by H_0 and the present time corresponds to $t_0 = 0.96$. The left bottom panel represents the evolution of energy density ρ versus redshift z . The solid lines correspond to energy density of Galileon phantom field with exponential potential for various values of β whereas dotted and dashed lines represent the energy density of matter and total energy density of the Universe respectively, for standard phantom field. At late times, the energy density of field gets to the matter, overtakes it and begins increasing ($w < -1$), and acquires the present accelerated expansion of the Universe. Afterwards ρ_ϕ continuously blows up, in future, after a finite interval of time. The right bottom panel shows the evolution of equation of state w versus redshift z for Galileon phantom field with steep exponential potential. It has another type of future singularity than the less steeper potential (linear potential) and continuously blows up to $-\infty$ after definite interval of redshift. For larger values of β , the Big Rip singularity is delayed in distant future. In this figure, all the plots have $V_0 = 1.2$ and $\beta = 0, 0.5, 1, 5, 10$ from top to bottom whereas bottom right plot from bottom to top.

For the case of $\beta = 0$, the Galileon phantom field model becomes standard phantom field model. Here we consider an exponential potential which is more steeper than the linear one. In this potential, we obtain different type of future singularity. The scale factor $a(t)$ diverges in distant future after a finite interval of time as shown in the upper right panel of figure II. In exponential potentials, Hao and Li obtained an attractor solution having $w < -1$ forming the “Big Rip” unavoidable [41]. Our numerical analysis shows that the energy density of phantom field blows up after finite interval of time and correspondingly the equation of state parameter w blows up to $-\infty$ (see bottom panels of figure II). This type of singularity has been discussed in different models, namely, brane worlds [42], Gauss-Bonnet cosmology [43] and tachyonic field [44]. For higher values of β , the Big Rip singularity is shifted in distant future. One can say that the Big Rip singularity is delayed for $\beta > 0$.

IV. CONCLUSION

In this paper, we have investigated cosmological dynamics of phantom and non phantom fields in presence of higher derivative Galileon correction \mathcal{L}_3 . For generality, we also studied phantom field with a general potential in order to check the impact of Galileon term on the structure of singularity.

In case of $\beta = 0$, the Galileon field (with linear potential) reduces to standard quintessence. In this case, as field evolves to the region of negative values of the potential, after finite interval of time (in future), the scale factor collapses giving rise to a Big Crunch singularity. To this effect, energy density of Galileon field ρ_ϕ shows collapsing nature in future. In case of standard Galileon field with linear potential, the Big Crunch singularity can be delayed depending upon the numerical values of β such that for large values of the parameter, delay may be considerable making the singularity practically redundant (see figure 1).

As for the phantom field, there are three types of singularities depending upon the nature of potential. In case of $V \sim \phi^n$, energy density diverges after infinite time for $n \leq 4$ whereas divergence is reached after finite time dubbed *Big Rip* if $n > 4$ including the case of exponential potential that corresponds to $n \rightarrow \infty$. In case of potentials steeper than the standard exponential, not only divergence of scale factor is reached in finite time but the equation of state parameter also diverges accordingly.

We have examined the probable future regimes of Universe with Galileon phantom field having a linear potential. In case of $\beta = 0$, the Galileon phantom field reduces to standard phantom. Due to negative sign in the kinetic term, the field rises up along the potential giving rise to singularity in future. The nature of this singularity is different for different type of potentials. In case of linear potential with Galileon phantom field, the equation of state parameter w approaches -1 with the slowly growing energy density compared to the standard case. For various values of β (0, 0.5, 1, 10, 100), we display our results in figure 2 which shows that the equation of state has less and less deviation from Λ CDM for larger values of β and asymptotically approaches -1 in distant future with the slowly increasing energy density as shown in the bottom panels of figure 2. In case of exponential potential that is more steeper than the linear one, it has different type of singularity, the equation of state blows up to $-\infty$ for a definite value of redshift and correspondingly the energy density ρ_ϕ diverges (see bottom panels of figure II) that is during a definite time an infinite energy density is reached and termed as “Big Rip” singularity which will rip galaxies apart some billion years before the actual Rip singularity is reached [45]. In this case, the larger values of β will delay the Big Rip singularity towards more and more distant future. We therefore conclude that in general the effect of Galileon correction to standard kinetic term in the Lagrangian generally results in the delayed approach to singularity. It might be interesting to investigate the behaviour of singularity using the full Galileon Lagrangian including \mathcal{L}_4 and \mathcal{L}_5 . Again, apart from the Big crunch or Big Rip singularities, it will be more interesting to study the effect of Galileon correction term on the other singularities like the pressure singularity or sudden singularity [46] and the softer type-IV singularity [47] which were extensively been studied in [48] and [49] and is deliberated to our future investigation.

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